Photon–meson transition form factors $\gamma\pi^0$, $\gamma\eta$ and $\gamma\eta'$ at low and moderately high Q^2

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(February 7, 2008)

We study photon–meson transition form factors $\gamma^*(Q^2)\gamma \to \pi^0, \eta, \eta'$ at low and moderately high virtualities Q^2 of one of the photons, the second photon being real. For the description of the form factor at low Q^2, a nontrivial quark–antiquark ($q\bar{q}$) structure of the photon in the soft region is assumed, i.e. the photon is treated much like an ordinary vector meson. At large Q^2, along with a perturbative tail of the soft wave function, as it is for a hadron, the photon wave function contains also a standard QED point-like $q\bar{q}$ component. The latter provides the $1/Q^2$ behavior of the transition form factor at large Q^2 in accordance with perturbative QCD. Using the experimental results on $\gamma\pi^0$ form factor, we reconstruct the soft photon wave function which is found to have the structure similar to pion's, which has been formerly determined from a study of the elastic pion form factor. Assuming the universality of the ground–state pseudoscalar meson wave functions we calculate the transition form factors $\gamma\eta$, $\gamma\eta'$ and partial widths $\eta \to \gamma\gamma$, $\eta' \to \gamma\gamma$, in a perfect agreement with data.

12.39.-x, 11.55.Fv, 12.38.Bx, 14.40.Aq

I. INTRODUCTION

In the present paper we continue the study of the transition regime from soft nonperturbative physics to the physics of hard processes described by the perturbative QCD (PQCD). In Ref. [1] we have proposed a method to consider a form factor in a broad range of momentum transfers starting from the nonperturbative region of small Q^2 and moving to moderately large Q^2 where form factor is represented as a series in α_s accounting for the nonperturbative term and $O(\alpha_s)$ corrections. This allows a continuous transition from small to asymptotically large momentum transfers. Within this procedure, we have determined the pion light–cone wave function by describing the pion elastic form factor in the range $0 \le Q^2 \le 10\ GeV^2$.

Recent experiments on pseudoscalar meson production in e^+e^- collisions [2] provide new data on the transition form factors $\gamma^*(Q^2)\gamma \to \pi^0, \eta, \eta'$ in the region of the momentum transfers $0 \le Q^2 \le 20\ GeV^2$. These results open a new possibility for studying the onset of the asymptotical PQCD regime. Reconstruction of the pseudoscalar meson wave function and photon $q\bar{q}$-distribution provides a bridge between the phenomenological soft-physics description and rigorous results of PQCD.

Theoretical investigation of the photon–meson transition processes which has a long history has given two important results on the photon–pion transition form factor:

(1) Adler–Bell–Jackiw axial anomaly [3] yields nonvanishing transition form factor of the pion into two real photons in the chiral limit of vanishing quark masses

$$F_{\gamma^*\gamma^*\pi}(Q_1^2 = 0,\ Q_2^2 = 0) = \frac{1}{2\sqrt{2}\pi^2 f_\pi}\ ,\ f_\pi = 130\ MeV\ , \tag{1}$$

where the photon–pion transition form factor is defined as follows ($Q_1^2 = -q_1^2\ ,\ Q_2^2 = -q_2^2$):

$$\begin{aligned}&\langle \pi(P)|T|\gamma(q_1,\mu)\gamma(q_2,\nu)\rangle \\ &\quad = e^2 \varepsilon_{\mu\nu\alpha_1\alpha_2} q_1^{\alpha_1} q_2^{\alpha_2} F_{\gamma^*\gamma^*\pi}(-q_1^2, -q_2^2).\end{aligned} \tag{2}$$

(2) In the kinematical region where at least one of the photon virtualities is large, PQCD gives the following prediction for the behavior of the transition form factor [4]

$$F_{\gamma^*\gamma^*\pi}(Q_1^2\ ,\ Q_2^2) = \frac{\sqrt{2}}{3}\int_0^1 \frac{\phi_\pi(x)dx}{xQ_1^2 + (1-x)Q_2^2}\ , \tag{3}$$

where $\phi_\pi(x)$ is the leading twist wave function (distribution amplitude) which describes the longitudinal momentum distribution of valence quark–antiquark pair in the pion. PQCD also predicts asymptotic behavior of the pion distribution amplitude in the following form [5]:

$$\phi_\pi^{as}(x) = 6 f_\pi x(1-x). \tag{4}$$

For $Q_2^2 \to 0$ and large $Q_1^2 \equiv Q^2$ which correspond to realistic kinematics of the experiments [2], Eq. (3) gives

$$F_{\gamma\pi}(Q^2) = \frac{\sqrt{2}}{3}\int_0^1 \frac{\phi_\pi(x)dx}{xQ^2}\left[1 + O(\alpha_s(Q^2))\right] + O\left(\frac{1}{Q^4}\right), \tag{5}$$

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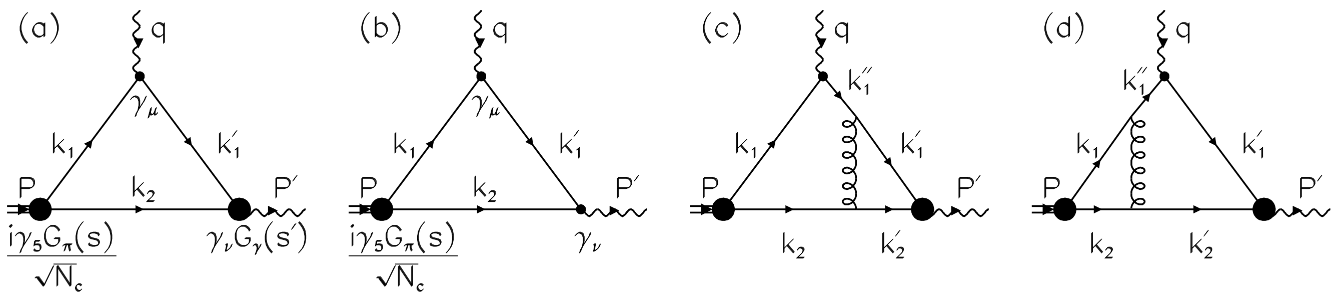


FIG. 1. Diagrams relevant to the description of the transition form factor at low and moderately high Q^2.

where $F_{\gamma\pi}(Q^2) \equiv F_{\gamma^*\gamma^*\pi}(Q^2, 0)$.

At asymptotically large Q^2 one can use the asymptotic pion distribution amplitude to find the leading behavior of the transition form factor

$$F_{\gamma\pi}(Q^2) = \frac{\sqrt{2} f_\pi}{Q^2} \left[1 + O(\alpha_s(Q^2)\right] + O\left(\frac{1}{Q^4}\right). \quad (6)$$

The leading term corresponds to the diagram of Fig. 1b with point-like vertices of the quark–photon interaction.

A problem starting from which Q^2 the form factor can be reliably described by the leading PQCD term only has been extensively discussed in connection with pion elastic form factor [6], [7]. As was firstly underlined in Ref. [6], asymptotical regime is switched on at considerably large momentum transfers, whereas in the region of $Q^2 \approx 10-20\ GeV^2$ the soft nonperturbative wave function gives a substantial contribution. Quantitative results for the pion form factor [1] agree with the statements of Ref. [6].

The procedure of Ref. [1] is the following: we divide the pion light–cone wave function Ψ^π into soft and hard components, Ψ^π_S and Ψ^π_H, such as Ψ^π_S is large at $s = (m^2 + \vec{k}^2_\perp)/(x(1-x)) < s_0$, while Ψ^π_H prevails at $s > s_0$. We performed this decomposition of the wave function using the step–function as a simplest ansatz

$$\Psi^\pi = \Psi^\pi_S \theta(s_0 - s) + \Psi^\pi_H \theta(s - s_0). \quad (7)$$

It is reasonable to represent the hard component, Ψ^π_H, as a series in α_s. Then at small and intermediate Q^2 the elastic pion form factor reads

$$F_\pi = F^{SS}_\pi + 2F^{SH}_\pi + O(\alpha_s^2), \quad (8)$$

where F^{SS}_π is a truly nonperturbative part of the form factor, and F^{SH}_π is an $O(\alpha_s)$ term with one–gluon exchange. The first term clearly dominates in the pion form factor at small Q^2. With minor corrections at small Q^2, the second term, as is known, provides the leading $\alpha_s(Q^2)/Q^2$ behavior of the elastic form factor at asymptotically large Q^2. So, the first two terms in the right-hand side of Eq. (8) accumulate the leading behavior from the two regions of small and large Q^2, thus allowing a realistic description in the region of intermediate momentum transfers.

Hard wave function, Ψ^π_H, is represented as a convolution of the one-gluon exchange kernel V^{α_s} with Ψ_S

$$\Psi^\pi_H = V^{\alpha_s} \otimes \Psi^\pi_S. \quad (9)$$

So, soft pion wave function Ψ^π_S is responsible for pion form factor behavior both at small and moderately large Q^2.

This strategy, with the incorporation of "photon wave function", is applicable for the description of the photon–pion transition form factor:

$$\Psi^\gamma = \Psi^\gamma_S \theta(s_0 - s) + \Psi^\gamma_H \theta(s - s_0). \quad (10)$$

However, an important distinction of the photon hard wave function compared with that of a hadron should be taken into account: namely, in addition to the hadronic component of the hard wave function which is related to the soft wave function via Eq. (9), the photon hard wave function contains also a standard point-like QED $q\bar{q}$–component such as

$$\Psi^\gamma_H = V^{\alpha_s} \otimes \Psi^\gamma_S + \Psi^\gamma_{pt}. \quad (11)$$

Corresponding expression for the form factor takes the form (Fig. 1):

$$F_{\gamma\pi} = \Psi^\pi_S \bullet \Psi^\gamma_S + \Psi^\pi_S \bullet \Psi^\gamma_H + \Psi^\pi_H \bullet \Psi^\gamma_S$$

$$= \Psi^\pi_S \bullet \Psi^\gamma_S + \Psi^\pi_S \bullet \Psi^\gamma_{pt} + \Psi^\pi_S \bullet V^{\alpha_s} \otimes \Psi^\gamma_S + \Psi^\pi_S \otimes V^{\alpha_s} \bullet \Psi^\gamma_S$$

$$\equiv F^{SS} + F^{Spt} + F^{SH(1)} + F^{SH(2)}\ . \quad (12)$$

Our analysis shows that at small momentum transfers the F^{SS} part dominates in the transition form factor, i.e. in the soft region photon should be treated much like an ordinary hadron, just in the spirit of the vector meson dominance. At large Q^2, the soft–point term (F^{Spt}), gives the leading $1/Q^2$ falloff, whereas the contribution of the $O(\alpha_s)$ terms ($F^{SH(1)} + F^{SH(2)}$) is suppressed by the additional factor α_s: the behavior of the photon–pion transition form factor differs from that of the elastic pion form factor where the soft–point term is absent and the soft–hard terms are dominant.

At intermediate momentum transfers, Eq. (12) provides substantial corrections to the $1/Q^2$ falloff: namely,

these corrections are due to the transverse motion of quarks in the soft–point term as well as to contributions related to the nontrivial $q\bar{q}$–structure of the soft photon (the terms involving Ψ_S^γ). The allowance for the corrections of the first type is a standard procedure in moving from asymptotically large to intermediate momentum transfers within the hard scattering approach [8] and modified hard scattering approach [9] which are based on the account for transverse motion and Sudakov effects [10]. However, as the present analysis shows, in the region of a few GeV^2 the contribution of the terms originated from the nontrivial $q\bar{q}$–structure of the soft photon is substantial and cannot be neglected.

The proposed method allows a self–consistent description of the photon–pion transition form factor in a broad range of Q^2. We use the soft pion wave function which has been previously determined in Ref. [1] by fitting elastic pion form factor. The description of the photon–pion transition form factor reveals a similarity of the low–s $q\bar{q}$-structure of soft photon with that of a pion.

Summing up, we obtain the following results:

1. We fit the available data on the photon–pion transition form factor at $Q^2 = 0-8\ GeV^2$ and determine the soft $q\bar{q}$ wave function of the photon. Pion wave function is taken from our description of elastic pion form factor [1]. Soft photon wave function turns out to be close to the ground–state meson wave function at low s, i.e. $\Psi_S^\gamma(s) \sim \Psi_S^\pi(s)$ at $s \le 2\ GeV^2$. The similarity of Ψ_S^γ and Ψ_S^π at $s \le 2\ GeV^2$ seems to be quite natural and corresponds to the vector meson dominance in the vertex $\gamma \to q\bar{q}$. At $s = s_0$ the photon wave function satisfies the boundary condition $\Psi_S^\gamma(s_0) = \Psi_{pt}^\gamma(s_0)$, which provides a correct sewing of the F^{SS} and F^{Spt} terms. Soft wave functions determined, we calculate the photon–pion transition form factor in a broad range of Q^2. Calculations show that several kinematical regions, where different contributions to the transition form factor dominate, may be isolated:

(i) At small $Q^2 = 0-5\ GeV^2$ the transition form factor is dominated by the soft–soft term which corresponds to nontrivial hadron–like structure of the soft photon.

(ii) At large $Q^2 \ge 50\ GeV^2$ the QED point-like component of the photon gives the main contribution reproducing the PQCD result.

(iii) In the intermediate region the transition form factor is an interplay of the soft–soft, soft–point and soft–hard contributions. Numerically, the form factor behavior is very close to the interpolation formula proposed by Brodsky and Lepage [4]:

$$F_{\gamma\pi}(Q^2) = \frac{\sqrt{2} f_\pi}{Q^2 + 4\pi^2 f_\pi^2}. \tag{13}$$

2. We calculate $\gamma\eta$ and $\gamma\eta'$ transition form factors, assuming universality of the $\vec{k}$-dependence of wave functions of all the ground–state pseudoscalar mesons where $\vec{k}$ is relative momentum of constituent quarks, $s = 4m^2 + 4\vec{k}^2$. This assumption is in line with the conventional quark model. In accordance with vector meson dominance, the same ansatz is used for relating the nonstrange and strange components of the soft photon. Then $\gamma\eta$ and $\gamma\eta'$ form factors are calculated with no free parameters. The results for the $\gamma\eta$ and $\gamma\eta'$ transition form factors are in an excellent agreement with the experimental data both on the shape of the form factors and on the decay partial widths, $\Gamma(\eta \to \gamma\gamma)$ and $\Gamma(\eta' \to \gamma\gamma)$. It provides an argument for small admixture of a glueball (two gluon) component into η and η': within the experimental accuracy we estimate corresponding probabilities as $W_\eta(glueball) < 10\%$ and $W_{\eta'}(glueball) < 20\%$.

The paper is organized as follows:
In Section II the calculation of the $\gamma\pi^0$ transition form factor is performed and the soft transition vertex $\gamma \to q\bar{q}$ is reconstructed. Section III is devoted to the calculation of $\gamma\eta$ and $\gamma\eta'$ transition form factors. Conclusive remarks are given in Section IV. Appendix presents the calculation details.

II. $\gamma\pi^0$ TRANSITION FORM FACTOR

We consider the $\gamma\pi^0$ transition form factor using the method presented in detail in Ref. [1]. So we omit here a discussion of the basic points of the technique, outlining only a skeleton of the calculation procedure.

The form factor $F_{\gamma\pi}$ is connected with the amplitude of the process $\gamma^*\gamma \to \pi^0$ as follows:

$$T_{\mu\nu}^{\gamma^*\gamma\pi}(q^2) = e^2 \varepsilon_{\mu\mu\alpha\beta} q^\alpha P^\beta F_{\gamma\pi}(-q^2), \tag{14}$$

where P is the pion momentum. Then partial width of the decay $\pi^0 \to \gamma\gamma$ reads

$$\Gamma(\pi^0 \to \gamma\gamma) = \frac{\pi}{4}\alpha^2 m_\pi^3 F_{\gamma\pi}^2(0), \quad \alpha = 1/137. \tag{15}$$

A. Soft–soft term $F_{\gamma\pi}^{SS}(Q^2)$

$F_{\gamma\pi}^{SS}(Q^2)$ corresponds to the diagram of Fig. 1a. The vertices of the soft transitions $\pi \to q\bar{q}$ and $q\bar{q} \to \gamma$ are defined as

$$\frac{\bar{q} i\gamma_5 q}{\sqrt{N_c}} G_\pi(s) \tag{16}$$

for $pion \to quark + antiquark$ (N_c is the number of colors) and

$$e_q \bar{q}\gamma_\nu q G_\gamma(s') \tag{17}$$

for $q\bar{q} \to photon$ (e_q is the quark charge).

The contribution of the diagram Fig. 1a can be written as the following double dispersion representation:

$$F^{SS}_{\gamma\pi}(Q^2) = Z_\pi 2 f_q(Q^2)\sqrt{N_c}\int \frac{ds G_\pi(s)\theta(s_0-s)}{\pi(s-m_\pi^2)} \times \frac{ds' G_\gamma(s')\theta(s_0-s')}{\pi s'}\Delta_{\pi\gamma^*\gamma}(s,s',Q^2). \quad (18)$$

Here $f_q(Q^2)$ is quark form factor, Z_π is a charge factor: $Z_\pi = (e_u^2 - e_d^2)/\sqrt{2}$, with $e_u = 2/3$ and $e_d = -1/3$.

The quantity $\Delta_{\pi\gamma^*\gamma}(s,s',Q^2)$ is the double spectral density of Feynman diagram of Fig. 1a with point-like vertices and the off–shell pion and photon momenta:

$$\varepsilon_{\mu\mu\alpha\beta} q^\alpha P^\beta \Delta_{\pi\gamma^*\gamma}(s,s',Q^2) = -disc_s disc_{s'} \int \frac{d^4k_2}{i(2\pi)^4}\frac{Sp\left(i\gamma_5(m-\hat{k}_2)\gamma_\nu(m+\hat{k}'_1)\gamma_\mu(m+\hat{k}_1)\right)}{(m^2-k_1^2)(m^2-k_2^2)(m^2-k_1'^2)},$$
$$s=(k_1+k_2)^2\ , \quad s'=(k'_1+k_2)^2\ . \quad (19)$$

m is the non–strange quark mass.

The trace reads:

$$Sp\left(i\gamma_5(m-\hat{k}_2)\gamma_\nu(m+\hat{k}'_1)\ \gamma_\mu\ (m+\hat{k}_1)\right) = -4m\varepsilon_{\mu\mu\alpha\beta}q^\alpha P^\beta. \quad (20)$$

Then one finds

$$\varepsilon_{\mu\mu\alpha\beta} q^\alpha P^\beta \Delta_{\pi\gamma^*\gamma}(s,s',Q^2) = \frac{(-i)^2(2\pi i)^3}{4} 4m\varepsilon_{\mu\mu\alpha\beta}q^\alpha P^\beta \times \int \frac{d^4k_2}{i(2\pi)^4}\delta(m^2-k_1^2)\delta(m^2-k_2^2)\delta(m^2-k_1'^2)\ ,$$
$$s=(k_1+k_2)^2\ , \quad s'=(k'_1+k_2)^2\ , \quad (21)$$

where

$$\Delta_{\pi\gamma^*\gamma}(s,s',Q^2) = \frac{m}{4}\frac{\theta(s'sQ^2 - m^2\lambda(s,s',Q^2))}{\lambda^{1/2}(s,s',Q^2)}, \quad (22)$$

and

$$\lambda(s,s',Q^2) = (s'-s)^2 + 2Q^2(s'+s) + Q^4. \quad (23)$$

Eqs. (18), (22) and (23) present $F^{SS}_{\gamma\pi}(Q^2)$ in terms of invariant variables s and s'. In the soft–soft term these variables are constrained within the intervals $4m^2 \le s \le s_0$ and $4m^2 \le s' \le s_0$.

Introducing the light–cone variables, $x = k_{2+}/P_+$, $\vec{k}_\perp = \vec{k}_{2\perp}$ and $q = (0, q_-, \vec{Q})$ (see Eq. (A6)), one finds

$$\Delta_{\pi\gamma^*\gamma}(s,s',Q^2) = \frac{m}{4\pi}\int \frac{dx d^2\vec{k}_\perp}{x(1-x)^2} \times \delta\left(s - \frac{m^2+\vec{k}_\perp^2}{x(1-x)}\right)\delta\left(s' - \frac{m^2+(\vec{k}_\perp - x\vec{Q})^2}{x(1-x)}\right) \quad (24)$$

Using these variables one comes to the following expression for the soft–soft form factor

$$F^{SS}_{\gamma\pi}(Q^2) = 2Z_\pi f_q(Q^2)\sqrt{N_c}\frac{m}{4\pi^5}\int \frac{dx d^2\vec{k}_\perp}{x(1-x)^2} \times \Psi_\pi(s)\theta(s_0-s)\frac{G_\gamma(s')}{s'}\theta(s_0-s'), \quad (25)$$

where

$$\Psi_\pi(s) = \frac{G_\pi(s)}{s-m_\pi^2} \quad (26)$$

and

$$s = \frac{m^2+\vec{k}_\perp^2}{x(1-x)},\ s' = \frac{m^2+(\vec{k}_\perp - x\vec{Q})^2}{x(1-x)}. \quad (27)$$

The value $F^{SS}_{\gamma\pi}(0)$ is connected with the known $\pi^0 \to \gamma\gamma$ decay width through Eq. (15). On the other hand, one finds

$$F^{SS}_{\gamma\pi}(0) = Z_\pi \frac{m}{2\pi}\int\limits_{4m^2}^{s_0} \frac{ds}{\pi}\Psi_\pi(s)\frac{G_\gamma(s)}{s} ln\frac{1+\sqrt{1-4m^2/s}}{1-\sqrt{1-4m^2/s}}. \quad (28)$$

We use this equation as a normalization condition for $G_\gamma(s)$.

$F^{SS}_{\gamma\pi}$ involves the constituent quark form factor which satisfies the condition $f_q(0) = 1$ and turns into Sudakov's form factor at large Q^2. The Sudakov form factor is taken in the form

$$S(Q^2) = exp\left(-\frac{\alpha_s(Q^2)}{2\pi}C_F ln^2\left(\frac{Q^2}{Q_0^2}\right)\right),\ C_F = \frac{N_c^2-1}{2N_c}, \quad (29)$$

where Q_0 is of the order of 1 GeV; we put $Q_0 = 1\ GeV$. Coupling constant $\alpha_s(Q^2)$ is assumed to be frozen below 1 GeV^2, namely we set

$$\alpha_s(Q^2) = \begin{cases} const\ , & Q < 1\ GeV\ , \\ \frac{4\pi}{9} ln^{-1}\left(\frac{Q^2}{\Lambda^2}\right)\ , & Q > 1\ GeV\ , \end{cases} \quad (30)$$

where $\Lambda = 220\ MeV$.

So, the constituent quark form factor is taken as

$$f_q(Q^2) = \begin{cases} 1\ , & Q < Q_0\ , \\ S(Q^2)\ , & Q > Q_0\ . \end{cases} \quad (31)$$

Let us stress that we have used here the same quark form factor as in Ref. [1].

B. Soft–point term $F^{Spt}_{\gamma\pi}(Q^2)$

Contribution from the diagram of Fig. 1b with $s' \ge s_0$ is denoted as $F^{Spt}_{\gamma\pi}(Q^2)$ and equals to:

$$F^{Spt}_{\gamma\pi}(Q^2) = 2Z_\pi f_q(Q^2)\sqrt{N_c}\frac{m}{4} \times \int \frac{ds}{\pi}\frac{ds'}{\pi s'}\Psi_\pi(s)\frac{\theta(s'sQ^2 - m^2\lambda(s,s',Q^2))}{\lambda^{1/2}(s,s',Q^2)},$$
$$4m^2 \le\ \ s \le s_0, \quad s' \ge s_0. \quad (32)$$

In the used normalization of the vertex, the point-like interaction is defined as $e_q\bar{q}\gamma_\nu q$, so the sewing of the soft–soft and soft–point terms requires

$$G_\gamma(s_0) = 1 \ . \tag{33}$$

Notice that $F^{Spt}_{\gamma\pi}(0) = 0$ which is the consequence of the constraints $s \le s_0$ and $s' \ge s_0$.

In terms of light–cone variables the expression for the $F^{Spt}_{\gamma\pi}(Q^2)$ reads

$$F^{Spt}_{\gamma\pi}(Q^2) = 2Z_\pi f_q(Q^2)\sqrt{N_c}\frac{m}{4\pi^5}\int\frac{dxd^2\vec{k}_\perp}{x(1-x)^2}\Psi_\pi(s)\frac{1}{s'}$$
$$\times\theta\left(s_0 - \frac{m^2+\vec{k}_\perp^2}{x(1-x)}\right)\theta\left(\frac{m^2+(\vec{k}_\perp - x\vec{Q})^2}{x(1-x)} - s_0\right), \tag{34}$$

with s and s' given by Eq. (27).

C. Soft–hard term $F^{SH(1)}_{\gamma\pi}(Q^2)$

Soft–hard form factor $F^{SH(1)}_{\gamma\pi}(Q^2)$ is connected with the amplitude of the diagram of Fig. 1c which can be written as the following dispersion representation

$$T^{\ SH(1)}_{\ \mu\nu}(Q^2) = 2e^2Z_\pi\sqrt{N_c}\int\frac{dsds'ds''}{\pi\ \pi\ \pi}\Psi_\pi(s)\frac{G_\gamma(s')}{s'}\frac{1}{s''}$$
$$\times D^{(1)}_{\mu\nu}(s,s',s'',Q^2)\theta(s_0-s)\theta(s_0-s')\theta(s''-s_0). \tag{35}$$

In this expression $D^{(1)}_{\mu\nu}(s,s',s'',Q^2)$ is the spectral density of the diagram of Fig. 1c:

$$D^{(1)}_{\mu\nu}(s,s',s'',Q^2) = C_F(2\pi i)^5\frac{(-i)^3}{8}\int\frac{d^4k_2}{i(2\pi)^4}\frac{d^4k'_2}{i(2\pi)^4}$$
$$\times\frac{4\pi\alpha_s(t)}{m_G^2-t}Sp^{(1)}_{\mu\nu}\delta(m^2-k_1^2)\delta(m^2-k_2^2)\delta(m^2-k_1'^2)$$
$$\times\delta(m^2-k_2'^2)\delta(m^2-k_1''^2)\ ,$$
$$s=(k_1+k_2)^2,\ s'=(k'_1+k'_2)^2,\ s''=(k''_1+k_2)^2. \tag{36}$$

Here $t = (k'_2-k_2)^2$ and m_G is an effective gluon mass; we consider several choices for m_G.

$Sp^{(1)}_{\mu\nu}$ denotes the following trace:

$$Sp^{(1)}_{\mu\nu} = Sp\Big(i\gamma_5(m-\hat{k}_2)\gamma_\alpha(m-\hat{k}'_2)\gamma_\nu(m+\hat{k}'_1)\gamma_\alpha$$
$$\times(m+\hat{k}''_1)\gamma_\mu(m+\hat{k}_1)\Big)\ . \tag{37}$$

For a detailed calculation of the quantity $Sp^{(1)}_{\mu\nu}$ we refer to Appendix. Isolating the factor $\varepsilon_{\mu\mu\alpha\beta}q^\alpha P^\beta$ one finds

$$Sp^{(1)}_{\mu\nu} = \varepsilon_{\mu\mu\alpha\beta}q^\alpha P^\beta\cdot S(1)$$
$$S(1) = 4m\left[s' - 12b' + 4b'' + (s''-s+Q^2)(a'_1-a'_2)\right]. \tag{38}$$

Analytic expressions for a'_1, a'_2, b', b'' are given in Appendix (see Eqs. (A5), (A8) and (A12)).

Thus, one finds

$$D^{(1)}_{\mu\nu}(s,s',s'',Q^2) = \varepsilon_{\mu\mu\alpha\beta}q^\alpha P^\beta\cdot\Delta^{(1)}(s,s',s'',Q^2),$$

where

$$\Delta^{(1)}(s,s',s'',Q^2) = -4\pi^5C_F\int\frac{d^4k_2}{i(2\pi)^4}\frac{d^4k'_2}{i(2\pi)^4}\frac{4\pi\alpha_s(t)}{m_G^2-t}$$
$$\times S(1)\delta(m^2-k_1^2)\delta(m^2-k_2^2)\delta(m^2-k_1'^2)$$
$$\times\delta(m^2-k_2'^2)\delta(m^2-k_1''^2)\ . \tag{39}$$

Finally, the dispersion representation for the soft–hard form factor $F^{SH(1)}_{\gamma\pi}(Q^2)$ reads

$$F^{\ SH(1)}_{\ \gamma\pi}(Q^2) = 2Z_\pi\sqrt{N_c}\int\frac{dsds'ds''}{\pi\ \pi\ \pi}\Psi_\pi(s)\frac{G_\gamma(s')}{s'}\frac{1}{s''}$$
$$\times\Delta^{(1)}(s,s',s'',Q^2)\theta(s_0-s)\theta(s_0-s')\theta(s''-s_0). \tag{40}$$

Introducing light–cone variables $x = k_{2+}/P_+$, $\vec{k}_\perp = \vec{k}_{2\perp}$, $x' = k'_{2+}/P_+$, $\vec{k}'_\perp = \vec{k}'_{2\perp}$ (see also Eqs. (A6), (A7)), one has:

$$\Delta^{(1)}(s,s',s'',Q^2) = \frac{C_F}{256\pi^3}\int\frac{dxd^2\vec{k}_\perp}{x(1-x)^2}\frac{dx'd^2\vec{k}'_\perp}{x'(1-x')}$$
$$\times\frac{4\pi\alpha_s(t)}{m_G^2-t}S(1)\delta\left(s-\frac{m^2+\vec{k}_\perp^2}{x(1-x)}\right)\delta\left(s'-\frac{m^2+\vec{k}'^2_\perp}{x'(1-x')}\right)$$
$$\times\delta\left(s''-\frac{m^2+(\vec{k}_\perp-x\vec{Q})^2}{x(1-x)}\right). \tag{41}$$

Substituting Eq. (41) into Eq. (40) yields the expression of $F^{SH(1)}_{\gamma\pi}(Q^2)$ in terms of the light–cone variables:

$$F^{SH(1)}_{\gamma\pi}(Q^2) = \frac{2Z_\pi C_F\sqrt{N_c}}{(16\pi^3)^2}\int\frac{dxd^2\vec{k}_\perp}{x(1-x)^2}\frac{dx'd^2\vec{k}'_\perp}{x'(1-x')}$$
$$\times\Psi_\pi(s)\frac{G_\gamma(s')}{s'}\frac{1}{s''}\frac{4\pi\alpha_s(t)}{m_G^2-t}S(1)$$
$$\times\theta(s_0-s)\theta(s_0-s')\theta(s''-s_0), \tag{42}$$

where

$$s = \frac{m^2+\vec{k}_\perp^2}{x(1-x)}\ ,\qquad s' = \frac{m^2+(\vec{k}'_\perp-x'\vec{Q})^2}{x'(1-x')}\ ,$$
$$s'' = \frac{m^2+(\vec{k}_\perp-x\vec{Q})^2}{x(1-x)}, \tag{43}$$

$$t = -\frac{m^2(x'-x)^2+(x\vec{k}'_\perp-x'\vec{k}_\perp)^2}{x'x} \tag{44}$$

The light–cone expressions for a'_1, a'_2, b', b'' are rather simple (see Eqs. (A8), (A12)), and the function $S(1)$ takes the form

$$S(1) = 4m\left[s' - 12\left(\frac{(\vec{k}_\perp\vec{Q})(\vec{k}'_\perp\vec{Q})}{Q^2} - (\vec{k}_\perp\vec{k}'_\perp)\right)\right.$$
$$\left. +4\left(\frac{(\vec{k}'_\perp\vec{Q})^2}{Q^2} - \vec{k}'^2_\perp\right) - (s'' - s + Q^2)\frac{(\vec{k}'_\perp\vec{Q})}{Q^2}\right]. \quad (45)$$

D. Soft–hard term $F^{SH(2)}_{\gamma\pi}(Q^2)$

Calculation of the soft–hard form factor $F^{SH(2)}_{\gamma\pi}(Q^2)$ looks very much like the calculation of $F^{SH(1)}_{\gamma\pi}(Q^2)$: $F^{SH(2)}_{\gamma\pi}(Q^2)$ is connected with the amplitude of the diagram Fig. 1d which can be written as the following dispersion representation

$$T^{\ SH(2)}_{\mu\nu}(Q^2) =$$
$$2\ e^2 Z_\pi\sqrt{N_c}\int\frac{ds ds' ds''}{\pi\ \pi\ \pi}\Psi_\pi(s)\frac{G_\gamma(s')}{s'}\frac{1}{s''-m_\pi^2}$$
$$\times D^{(2)}_{\mu\nu}(s,s',s'',Q^2)\theta(s_0-s)\theta(s_0-s')\theta(s''-s_0). \quad (46)$$

Here $D^{(2)}_{\mu\nu}(s,s',s'',Q^2)$ is the spectral density of the diagram of Fig. 1d which reads

$$D^{(2)}_{\mu\nu}(s,s',s'',Q^2) = C_F(2\pi i)^5\frac{(-i)^3}{8}\int\frac{d^4k_2}{i(2\pi)^4}\frac{d^4k'_2}{i(2\pi)^4}$$
$$\times\frac{4\pi\alpha_s(t)}{m_G^2-t}Sp^{(2)}_{\mu\nu}\delta(m^2-k_1^2)\delta(m^2-k_2^2)\delta(m^2-k'^2_1)$$
$$\times\delta(m^2-k'^2_2)\delta(m^2-k''^2_1)\ ,$$
$$s=(k_1+k_2)^2,\ s'=(k'_1+k'_2)^2,\ s''=(k''_1+k'_2)^2. \quad (47)$$

$Sp^{(2)}_{\mu\nu}$ denotes the trace

$$Sp^{(2)}_{\mu\nu} = Sp\left(i\gamma_5(m-\hat{k}_2)\gamma_\alpha(m-\hat{k}'_2)\gamma_\nu(m+\hat{k}'_1)\gamma_\mu\right.$$
$$\left.\times(m+\hat{k}''_1)\gamma_\alpha(m+\hat{k}_1)\right). \quad (48)$$

A detailed calculation of the quantity $Sp^{(2)}_{\mu\nu}$ is presented in Appendix. Isolating the factor $\varepsilon_{\mu\mu\alpha\beta}q^\alpha P^\beta$ one finds

$$Sp^{(2)}_{\mu\nu} = \varepsilon_{\mu\mu\alpha\beta}q^\alpha P^\beta\cdot S(2)$$
$$S(2) = 8m\left[s - (a'_1-a'_2)(k''_1P) - a'_2(k'_1k'_2) - 2b''\right.$$
$$\left. -m^2(1+a'_2)\right]. \quad (49)$$

The expressions for a'_1, a'_2, b', b'' are given in Appendix (Eqs. (A5), (A8) and (A12)).

So,

$$D^{(2)}_{\mu\nu}(s,s',s'',Q^2) = \varepsilon_{\mu\mu\alpha\beta}q^\alpha P^\beta\cdot\Delta^{(2)}(s,s',s'',Q^2)$$
$$\Delta^{(2)}(s,s',s'',Q^2) = -4\pi^5 C_F\int\frac{d^4k_2}{i(2\pi)^4}\frac{d^4k'_2}{i(2\pi)^4}\frac{4\pi\alpha_s(t)}{m_G^2-t}$$
$$\times S(2)\delta(m^2-k_1^2)\delta(m^2-k_2^2)\delta(m^2-k'^2_1)$$
$$\times\delta(m^2-k'^2_2)\delta(m^2-k''^2_1)\ . \quad (50)$$

Finally, the soft–hard form factor $F^{SH(2)}_{\gamma\pi}(Q^2)$ reads

$$F^{\ SH(2)}_{\gamma\pi}(Q^2) =$$
$$2\ Z_\pi\sqrt{N_c}\int\frac{ds ds' ds''}{\pi\ \pi\ \pi}\Psi_\pi(s)\frac{G_\gamma(s')}{s'}\frac{1}{s''-m_\pi^2}$$
$$\times\Delta^{(2)}(s,s',s'',Q^2)\theta(s_0-s)\theta(s_0-s')\theta(s''-s_0). \quad (51)$$

Using light–cone variables one gets

$$\Delta^{\ (2)}(s,s',s'',Q^2) = \frac{C_F}{256\pi^3}\int\frac{dx d^2\vec{k}_\perp}{x(1-x)}\frac{dx' d^2\vec{k}'_\perp}{x'(1-x')^2}$$
$$\times\frac{4\pi\alpha_s(t)}{m_G^2-t}S(2)\delta\left(s-\frac{m^2+\vec{k}^2_\perp}{x(1-x)}\right)\delta\left(s'-\frac{m^2+\vec{k}'^2_\perp}{x'(1-x')}\right)$$
$$\times\delta\left(s''-\frac{m^2+(\vec{k}'_\perp+x'\vec{Q})^2}{x'(1-x')}\right). \quad (52)$$

Substituting Eq. (52) into Eq. (51) leads to the expression for $F^{SH(2)}_{\gamma\pi}(Q^2)$ in terms of the light–cone variables:

$$F^{SH(2)}_{\gamma\pi}(Q^2) = \frac{2Z_\pi C_F\sqrt{N_c}}{(16\pi^3)^2}\int\frac{dx d^2\vec{k}_\perp}{x(1-x)}\frac{dx' d^2\vec{k}'_\perp}{x'(1-x')^2}$$
$$\times\Psi_\pi(s)\frac{G_\gamma(s')}{s'}\frac{1}{s''-m_\pi^2}\frac{4\pi\alpha_s(t)}{m_G^2-t}S(2)$$
$$\times\theta(s_0-s)\theta(s_0-s')\theta(s''-s_0), \quad (53)$$

where

$$s=\frac{m^2+\vec{k}^2_\perp}{x(1-x)}, s'=\frac{m^2+(\vec{k}'_\perp-x'\vec{Q})^2}{x'(1-x')}, s''=\frac{m^2+\vec{k}'^2_\perp}{x'(1-x')}. \quad (54)$$

Using the light–cone expressions for a'_1, a'_2, b', b'' (see Eqs. (A8), (A12)), one gets the following result for $S(2)$:

$$S(2) = 8m\left[s + \frac{(\vec{k}'_\perp\vec{Q})}{Q^2}(k''_1P) - \left(x' + \frac{(\vec{k}'_\perp\vec{Q})}{Q^2}\right)(k'_1k'_2)\right.$$
$$\left. -2\left(\frac{(\vec{k}'_\perp\vec{Q})^2}{Q^2} - \vec{k}'^2_\perp\right) - m^2\left(1 + x' + \frac{(\vec{k}'_\perp\vec{Q})}{Q^2}\right)\right]. \quad (55)$$

E. Calculation results

Calculations of form factors at low and intermediate Q^2 include masses which are related to the soft physics: constituent quark mass and effective gluon mass. We use standard constituent quark mass $m = 0.35\ GeV$ while for effective gluon mass, m_G, we consider, just as in Ref. [1], the following two variants (masses are given in GeV):

$$(i)\quad m_G = 0\ ,$$
$$(ii)\quad m_G = \begin{cases} 0.7\left(1+\frac{t}{0.5}\right) & ,\ |t| < 0.5\ GeV^2\ , \\ 0 & ,\ |t| > 0.5\ GeV^2\ . \end{cases} \quad (56)$$

For $\alpha_s(t)$ we use two parametrization:

$$(i) \quad \alpha_s(t) = \begin{cases} const & , \ |t| < 1 \ GeV^2 \ , \\ \frac{4\pi}{9} ln^{-1}\left(\frac{|t|}{\Lambda^2}\right) & , \ |t| > 1 \ GeV^2 \ , \end{cases}$$

$$(ii) \quad \alpha_s(t) = \alpha_s\left(\frac{Q^2}{4}\right). \qquad (57)$$

The first variant corresponds to a frozen $\alpha_s(t)$ at low t: it is the same variant as in Eq. (31). In the second variant we take $\alpha_s(t)$ in the middle point, $|t| \to Q^2/4$. So, Eqs. (56) and (57) provide four possible variants for the soft–hard form factors. Corresponding results for soft–hard term are shown in Fig. 2. The main conclusion coming from the study of these variants is that a concrete choice of m_G and α_S in the soft region does not much influence the result.

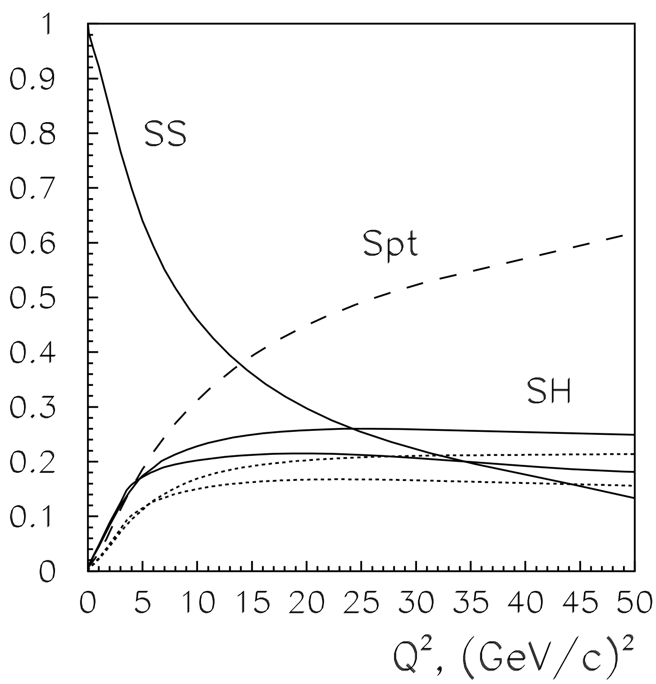


FIG. 2. Relative contributions of different terms to the form factor: the ratios of different terms contributing to the whole form factor are shown. Four variants for the SH–term correspond to: (1) $m_G = 0$, $\alpha_s(t)$ – the upper solid curve; (2) $m_G = 0$, $\alpha_s\left(\frac{Q^2}{4}\right)$ – the lower solid curve; (3) $m_G = m_G(t)$, $\alpha_s(t)$ – the upper dotted curve; (4) $m_G = m_G(t)$, $\alpha_s\left(\frac{Q^2}{4}\right)$ – the lower dotted curve.

Much more important is a choice of the soft photon wave function. There are two constraints on the soft photon wave function: the normalization condition (28) which guarantees the correct decay widht $\pi^0 \to \gamma\gamma$, and the sewing condition of Eq. (33). In addition, we impose the third constraint in the spirit of vector dominance model and $SU(6)$–symmetry: the slope of $G_\gamma(s)$ in the region $4m^2 < s < 1.5 \ GeV^2$ is the same as that of $\pi(s)$. Using these constraints we reconstruct the photon $q\bar{q}$–distribution function $G_\gamma(s)$ which is shown in Fig. 3, together with $G_\pi(s)$ for the comparison. The reconstructed distribution function has a dip in the region $s \sim 2-4 \ GeV^2$ similar to that of $G_\pi(s)$. In Ref. [1] the $q\bar{q}$–distribution in the pion is referred as quasizone one. The reconstructed $G_\gamma(s)$ behaviour provides an argument that the quasizone structure in the $q\bar{q}$–distributions is of a common nature. The quantity $\frac{\pi}{4}\alpha^2 m_\pi^3 F^2_{\gamma\pi}(Q^2)$ with the reconstructed $G_\gamma(s)$ is presented in Fig. 4.

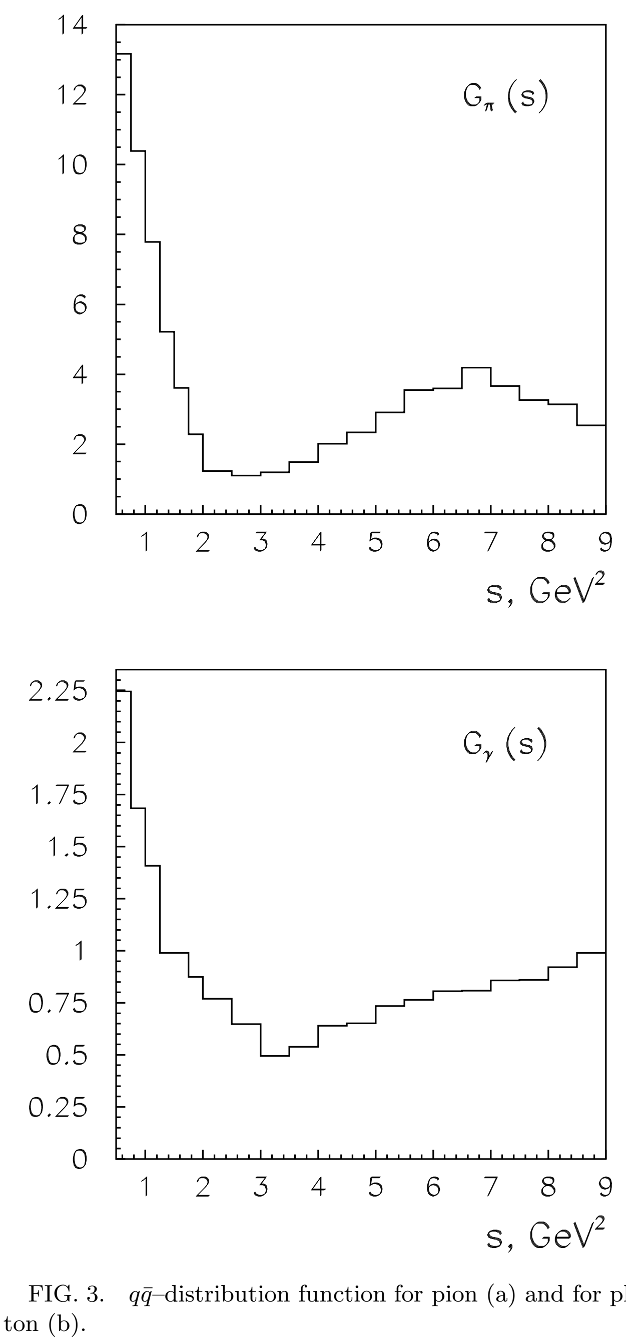


FIG. 3. $q\bar{q}$–distribution function for pion (a) and for photon (b).

To check the form factor sensitivity to the $q\bar{q}$–

distribution function we have calculated $F_{\gamma\pi}(Q^2)$ with alternative choices of $G_\gamma(s)$:

$$(1) \quad G_\gamma^{(1)}(s) = \begin{cases} 0.98 G_\gamma(s) & , \; s \le 1.5\ GeV^2 \ , \\ 1 & , \; s > 1.5\ GeV^2 \ , \end{cases} \ ,$$

$$(2) \quad G_\gamma^{(2)}(s) = 1 \ . \tag{58}$$

The results are shown in Fig. 5. The variant (1) corresponds to the low–s $q\bar{q}$–distribution similar to $G_\pi(s)$ but without a dip at $s \sim 3\ GeV^2$ (the factor 0.98 is due to renormalization of the $q\bar{q}$–distribution to have $\Gamma_{\gamma\gamma} = 2.23\ eV$). The absence of a dip in the photon $q\bar{q}$–distribution results in raising the calculated curve at large Q^2. The variant (2), which corresponds to the point-like vertex $q\bar{q} \to photon$ at low s, represents neither $\Gamma_{\gamma\gamma}$ nor low–Q^2 form factor correctly.

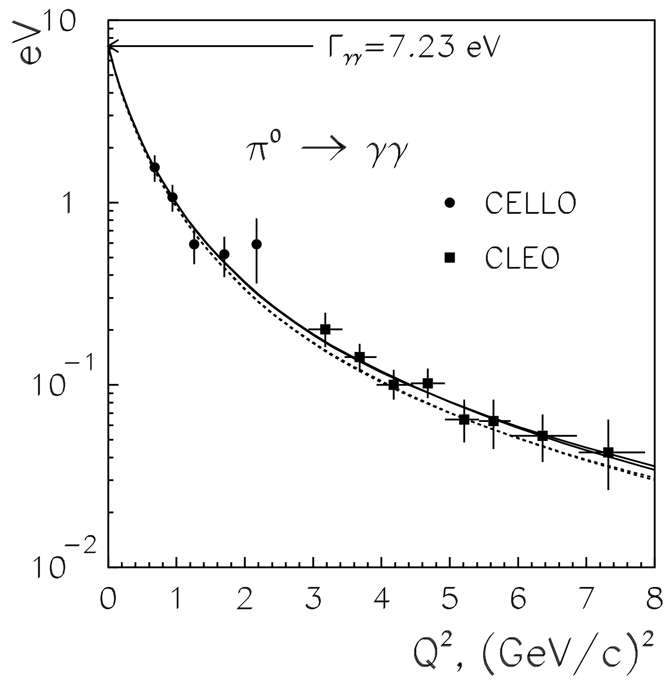


FIG. 4. $\gamma\pi^0$ transition form factor: the quantity $\frac{\pi}{4}\alpha^2 m_\pi^3 F_{\gamma\pi}^2(Q^2)$ is shown. Different curves correspond to different sets of parameters in the SH-term; the curve notation is the same as in Fig. 2. Experimental data are taken from Ref. [2].

III. $\gamma\eta$ AND $\gamma\eta'$ TRANSITION FORM FACTORS

In our calculations of the $\gamma\eta$ and $\gamma\eta'$ transition form factors we assume, in the spirit of the quark model, the universality of soft wave functions of the 0^-–nonet. This universality is usually formulated in the r–representation, or in terms of the relative quark momenta. So, let us rewrite the pion wave function $\Psi_\pi(s)$ (see Eq. (26)) as a function of $\vec{k}$ which is connected with the energy squared by $\vec{k}^2 = (s - 4m^2)/4$.

For the momentum representation of the pion wave function we use the following form

$$\Psi_\pi(s) = \psi_\pi(\vec{k}^2) = \frac{g(\vec{k}^2)}{\vec{k}^2 + k_0^2} \ , \tag{59}$$

where according to Ref. [1] $k_0^2 = 0.1176$ GeV^2.

The pseudoscalar mesons η and η' are mixtures of the non-strange and strange quarks, $n\bar{n} = (u\bar{u} + d\bar{d})/\sqrt{2}$ and $s\bar{s}$: $\eta = n\bar{n}\ Cos\theta - s\bar{s}\ Sin\theta$, $\eta' = n\bar{n}\ Sin\theta + s\bar{s}\ Cos\theta$. Respectively, the wave functions of the η and η' mesons are described by the two components

$$\Psi_\eta = Cos\theta\ \psi_{n\bar{n}}(\vec{k}^2) - Sin\theta\ \psi_{s\bar{s}}(\vec{k}^2) \ ,$$
$$\Psi_{\eta'} = Sin\theta\ \psi_{n\bar{n}}(\vec{k}^2) + Cos\theta\ \psi_{s\bar{s}}(\vec{k}^2) \ . \tag{60}$$

The universality of the pseudoscalar meson wave function means that the function

$$\psi_{n\bar{n}}(\vec{k}^2) = \frac{g(\vec{k}^2)}{\vec{k}^2 + k_0^2} \ , \tag{61}$$

normalized as follows

$$\frac{2}{\pi^2} \int\limits_0^{(s_0 - 4m^2)/4} d\vec{k}^2\ k\sqrt{\vec{k}^2 + m^2}\,\psi_{n\bar{n}}^2(\vec{k}^2) = 1, \tag{62}$$

is just the same function as for the π–meson.

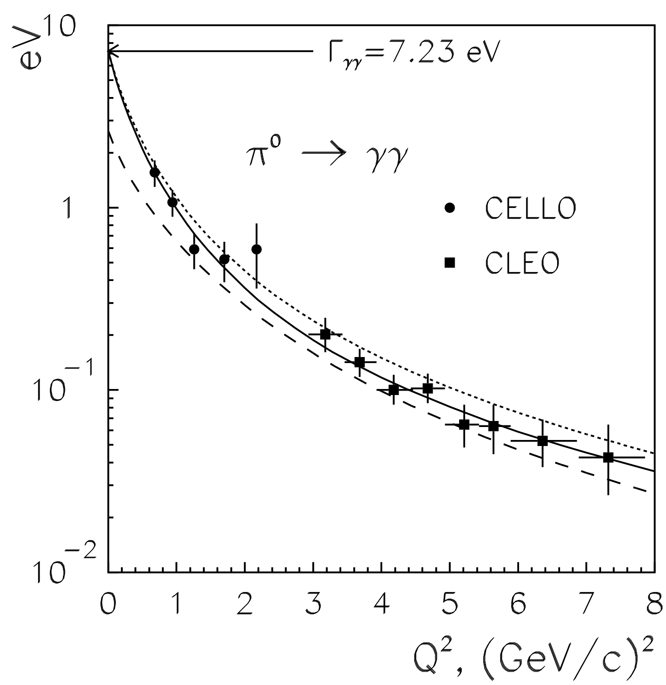


FIG. 5. The quantity $\frac{\pi}{4}\alpha^2 m_\pi^3 F_{\gamma\pi}^2(Q^2)$ for the variant (1) of Eq. (57) – dotted line, and for the variant (2) – dashed line. The solig line is the same as in Fig. 4.

For the $s\bar{s}$–component we should take into account strange/non–strange quark mass difference, hence

$$\psi_{s\bar{s}}(\vec{k}^2) = N\frac{g(\vec{k}^2)}{\vec{k}^2 + k_0^2 + \Delta^2}\ , \tag{63}$$

where $\Delta^2 = m_s^2 - m^2$, $m_s = 0.5\ GeV$ and $\vec{k}^2 = (s-4m_s^2)/4$. Numerical factor N corresponds to a renormalization of $\psi_{s\bar{s}}(\vec{k}^2)$ after introducing Δ^2. The factor N for the strange component of the wave function is determined by the normalization condition

$$\frac{2}{\pi^2}\int\limits_0^{(s_0-4m^2)/4} d\vec{k}^2\ k\sqrt{\vec{k}^2+m_s^2}\psi^2_{s\bar{s}}(\vec{k}^2) = 1\ . \tag{64}$$

Similarly, for the momentum representation of the soft photon wave function we use the expressions

$$\Psi_\gamma(s) \equiv \psi_{\gamma\to n\bar{n}}(\vec{k}^2) = \frac{g_\gamma(\vec{k}^2)}{\vec{k}^2+m^2} \tag{65}$$

and

$$\psi_{\gamma\to s\bar{s}}(\vec{k}^2) = \frac{g_\gamma(\vec{k}^2)}{\vec{k}^2+m^2+\Delta^2}. \tag{66}$$

Thus, the $n\bar{n}$ and $s\bar{s}$ components of the meson and soft photon wave function being fixed, one can proceed with calculations of the transition $\gamma\eta$ and $\gamma\eta'$ form factors, which are determined as

$$\begin{aligned} F_{\gamma\eta}(Q^2) &= Cos\theta\ F_{n\bar{n}}(Q^2) - Sin\theta\ F_{s\bar{s}}(Q^2)\ , \\ F_{\gamma\eta'}(Q^2) &= Sin\theta\ F_{n\bar{n}}(Q^2) + Cos\theta\ F_{s\bar{s}}(Q^2)\ . \end{aligned} \tag{67}$$

In the next two subsections we present formulae for $F_{n\bar{n}}(Q^2)$ and $F_{s\bar{s}}(Q^2)$.

A. The $n\bar{n}$ contributions to the form factor

These contributions can be obtained from the corresponding terms in $F_{\gamma\pi}(Q^2)$ by replacing the charge factor and wave functions. The final results are listed below.

Soft–soft term $F^{SS}_{n\bar{n}}(Q^2)$:

$$\begin{aligned} F^{SS}_{n\bar{n}}(Q^2) &= 2Z_{n\bar{n}}f_q(Q^2)\sqrt{N_c}\frac{m}{4\pi^5}\int\frac{dxd^2\vec{k}_\perp}{x(1-x)^2} \\ &\times\psi_{n\bar{n}}(\vec{k}^2)\theta(s_0-s)\frac{g_\gamma(\vec{k}'^2)}{s'}\theta(s_0-s'), \end{aligned} \tag{68}$$

$$Z_{n\bar{n}} = \frac{e_u^2+e_d^2}{\sqrt{2}},\quad \vec{k}^2 = \frac{s}{4}-m^2,\quad \vec{k}'^2 = \frac{s'}{4}-m^2, \tag{69}$$

$$\begin{aligned} F^{SS}_{n\bar{n}}(0) &= Z_{n\bar{n}}\frac{m}{2\pi} \\ &\times\int\limits_{4m^2}^{s_0}\frac{ds}{\pi}\psi_{n\bar{n}}(\vec{k}^2)\frac{g_\gamma(\vec{k}^2)}{s}ln\frac{1+\sqrt{1-4m^2/s}}{1-\sqrt{1-4m^2/s}}. \end{aligned} \tag{70}$$

s and s' are given by Eq. (27).

Soft–point term $F^{Spt}_{n\bar{n}}(Q^2)$:

$$\begin{aligned} F^{SPt}_{n\bar{n}}(Q^2) &= 2Z_{n\bar{n}}f_q(Q^2)\sqrt{N_c}\frac{m}{4\pi^5}\int\frac{dxd^2\vec{k}_\perp}{x(1-x)^2} \\ &\times\psi_{n\bar{n}}(\vec{k}^2)\frac{1}{s'}\theta(s_0-s)\theta(s'-s_0). \end{aligned} \tag{71}$$

Soft–hard term $F^{SH(1)}_{n\bar{n}}(Q^2)$:

$$\begin{aligned} F^{SH(1)}_{n\bar{n}}(Q^2) &= \frac{2Z_{n\bar{n}}C_F\sqrt{N_c}}{(16\pi^3)^2}\int\frac{dxd^2\vec{k}_\perp}{x(1-x)^2}\frac{dx'd^2\vec{k}'_\perp}{x'(1-x')} \\ &\times\psi_{n\bar{n}}(\vec{k}^2)\frac{g_\gamma(\vec{k}'^2)}{s'}\frac{1}{s''}\frac{4\pi\alpha_s(t)}{m_G^2-t}S(1) \\ &\times\theta(s_0-s)\theta(s_0-s')\theta(s''-s_0), \end{aligned} \tag{72}$$

s, s' and s'' are given by Eq. (43).

Soft–hard term $F^{SH(2)}_{n\bar{n}}(Q^2)$:

$$\begin{aligned} F^{SH(2)}_{n\bar{n}}(Q^2) &= \frac{2Z_{n\bar{n}}C_F\sqrt{N_c}}{(16\pi^3)^2}\int\frac{dxd^2\vec{k}_\perp}{x(1-x)}\frac{dx'd^2\vec{k}'_\perp}{x'(1-x')^2} \\ &\times\psi_{n\bar{n}}(\vec{k}^2)\frac{g_\gamma(\vec{k}'^2)}{s'}\frac{1}{s''-m^2_{\eta,\eta'}}\frac{4\pi\alpha_s(t)}{m_G^2-t}S(2) \\ &\times\theta(s_0-s)\theta(s_0-s')\theta(s''-s_0), \end{aligned} \tag{73}$$

s, s' and s'' are given by Eq. (54).

B. The $s\bar{s}$ contributions to the form factor

These expressions are obtained from the corresponding terms in $F_{\gamma\pi}(Q^2)$ by replacing the charge factor, wave functions and quark masses. The results have the following form.

Soft–soft term $F^{SS}_{s\bar{s}}(Q^2)$:

$$\begin{aligned} F^{SS}_{s\bar{s}}(Q^2) &= 2Z_{s\bar{s}}f_q(Q^2)\sqrt{N_c}\frac{m_s}{4\pi^5}\int\frac{dxd^2\vec{k}_\perp}{x(1-x)^2} \\ &\times\psi_{s\bar{s}}(\vec{k}^2)\theta(s_0-s)\frac{g_\gamma(\vec{k}'^2)}{s'}\theta(s_0-s')\ , \end{aligned} \tag{74}$$

$$Z_{s\bar{s}} = e_s^2,\quad \vec{k}^2 = \frac{s}{4}-m_s^2,\quad \vec{k}'^2 = \frac{s'}{4}-m_s^2\ , \tag{75}$$

$$s = \frac{m_s^2+\vec{k}_\perp^2}{x(1-x)},\ s' = \frac{m_s^2+(\vec{k}_\perp-x\vec{Q})^2}{x(1-x)}. \tag{76}$$

$$\begin{aligned} F^{SS}_{s\bar{s}}(0) &= Z_{s\bar{s}}\frac{m_s}{2\pi} \\ &\times\int\limits_{4m_s^2}^{s_0+4\Delta^2}\frac{ds}{\pi}\psi_{s\bar{s}}(\vec{k}^2)\frac{g_\gamma(\vec{k}^2)}{s}ln\frac{1+\sqrt{1-4m_s^2/s}}{1-\sqrt{1-4m_s^2/s}} \end{aligned} \tag{77}$$

Soft–point term $F^{Spt}_{s\bar{s}}(Q^2)$:

$$F_{s\bar{s}}^{SPt}(Q^2) = 2Z_{s\bar{s}} f_q(Q^2)\sqrt{N_c}\frac{m_s}{4\pi^5}\int \frac{dxd^2\vec{k}_\perp}{x(1-x)^2} \times \psi_{s\bar{s}}(\vec{k}^2)\frac{1}{s'}\theta(s_0-s)\theta(s'-s_0). \tag{78}$$

Soft–hard term $F_{s\bar{s}}^{SH(1)}(Q^2)$:

$$F_{s\bar{s}}^{SH(1)}(Q^2) = \frac{2Z_{s\bar{s}}C_F\sqrt{N_c}}{(16\pi^3)^2}\int \frac{dxd^2\vec{k}_\perp}{x(1-x)^2}\frac{dx'd^2\vec{k}'_\perp}{x'(1-x')} \times \psi_{s\bar{s}}(\vec{k}^2)\frac{g_\gamma(\vec{k}'^2)}{s'}\frac{1}{s''}\frac{4\pi\alpha_s(t)}{m_G^2-t}S(1) \times \theta(s_0-s)\theta(s_0-s')\theta(s''-s_0), \tag{79}$$

$$s = \frac{m_s^2+\vec{k}_\perp^2}{x(1-x)} \quad , \quad s' = \frac{m_s^2+(\vec{k}'_\perp - x'\vec{Q})^2}{x'(1-x')} \ , \quad s'' = \frac{m_s^2+(\vec{k}_\perp - x\vec{Q})^2}{x(1-x)}, \tag{80}$$

$$t = -\frac{m_s^2(x'-x)^2+(x\vec{k}'_\perp - x'\vec{k}_\perp)^2}{x'x} \tag{81}$$

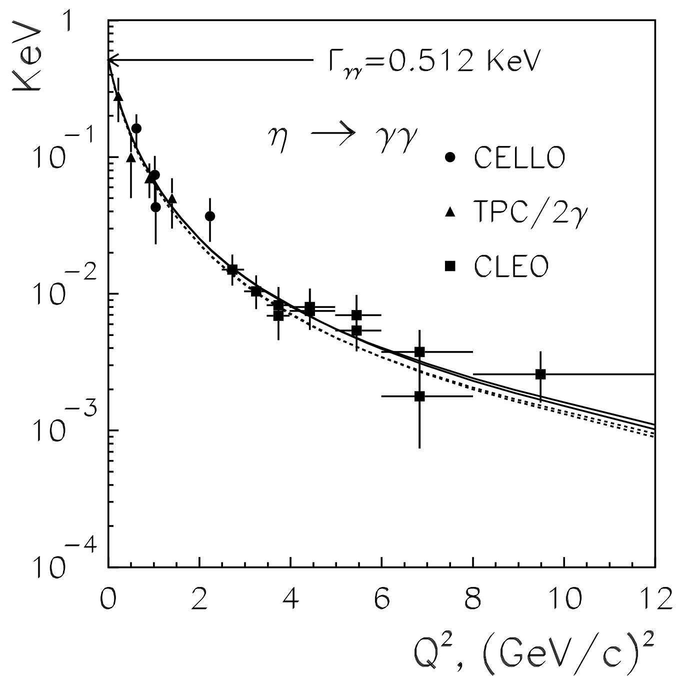
Soft–hard term $F_{s\bar{s}}^{SH(2)}(Q^2)$:

$$F_{s\bar{s}}^{SH(2)}(Q^2) = \frac{2Z_{s\bar{s}}C_F\sqrt{N_c}}{(16\pi^3)^2}\int \frac{dxd^2\vec{k}_\perp}{x(1-x)}\frac{dx'd^2\vec{k}'_\perp}{x'(1-x')^2} \times \psi_{s\bar{s}}(\vec{k}^2)\frac{g_\gamma(\vec{k}'^2)}{s'}\frac{1}{s''-m^2_{\eta,\eta'}}\frac{4\pi\alpha_s(t)}{m_G^2-t}S(2) \times \theta(s_0-s)\theta(s_0-s')\theta(s''-s_0), \tag{82}$$

$$s = \frac{m_s^2+\vec{k}_\perp^2}{x(1-x)}, s' = \frac{m_s^2+(\vec{k}'_\perp - x'\vec{Q})^2}{x'(1-x')}, s'' = \frac{m_s^2+\vec{k}'^2_\perp}{x'(1-x')}. \tag{83}$$

C. Calculation results for the $\gamma\eta$ and $\gamma\eta'$ transition form factors

Calculation of the $\gamma\eta$ and $\gamma\eta'$ transition form factors does not require any additional parameter, for all unknown quantities are fixed by the $\gamma\pi$ case. We use only the universality of wave functions of the ground–state pseudoscalar meson nonet; we put also $Sin\theta = 0.61$ [11]. The results show an excellent agreement with data both in shapes of the Q^2 dependence of form factors (see Fig. 6) and in absolute values of $F_{\gamma\eta}(0)$ and $F_{\gamma\eta'}(0)$: experimental data for the partial decay widths are $\Gamma_{\eta\to\gamma\gamma} = 0.514 \pm 0.052\ KeV$ [11] and $\Gamma_{\eta'\to\gamma\gamma} = 4.57 \pm 0.69\ KeV$ [11] while our calculation gives $\Gamma_{\eta\to\gamma\gamma} = 0.512\ KeV$ and $\Gamma_{\eta'\to\gamma\gamma} = 4.81\ KeV$. The universal wave functions are presented in Table I.



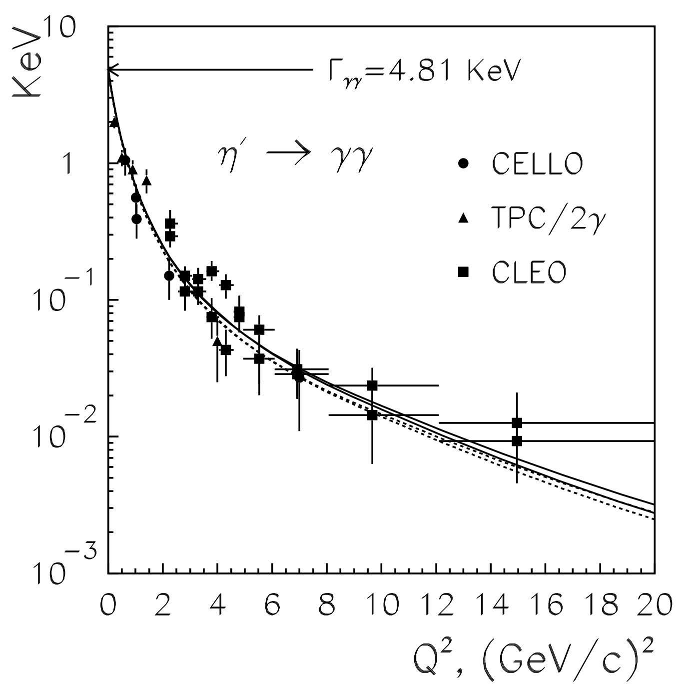


FIG. 6. Q^2-dependence for the quantities $\frac{\pi}{4}\alpha^2 m_\eta^3 F^2_{\gamma\eta}(Q^2)$ and $\frac{\pi}{4}\alpha^2 m_\eta'^3 F^2_{\gamma\eta'}(Q^2)$. Different curves correspond to different sets of the parameters in the SH-term (the curve notation is the same as in Fig. 2). Experimental data are taken from Ref. [2].

Within the hypothesis about universality of pseudoscalar meson wave functions, we may estimate the value of gluonic (or glueball) components in η and η'. With $\eta = C_1 q\bar{q} + C_2 gg$ and $\eta' = C'_1 q\bar{q} + C'_2 gg$, we obtain the following constraints for probabilities of the gluonic com-

ponents in η and η': $C_2^2 \le 0.1$ and $C_2'^2 \le 0.2$.

IV. CONCLUSION

We have analyzed the $\gamma\pi^0$, $\gamma\eta$, and $\gamma\eta'$ transition form factors at low and moderately high Q^2 in the framework of the method developed in Ref. [1] which allowes us to cover the range of momenta from the soft to PQCD physics. Calculation results back up the efficiency of the proposed method.

This consideration support the results of Ref. [1] where the pion wave function has been reconstructed. Here we have reconstructed the soft $q\bar{q}$–distribution of the photon: it has been found to be similar to the quark distribution in the pion, that is quite natural in the framework of the vector dominance model for soft $q\bar{q}$.

The calculated $\gamma\eta$ and $\gamma\eta'$ transition form factors are in a perfect agreement with the experimental data, once we assume the universality of the soft wave functions of the ground–state pseudoscalar mesons, the members of the lightest nonet. This assumptions looks very natural and inherent to the conventional quark model.

The transition photon–meson form factors at low Q^2 were calculated in Refs. [12], [13] using the diagram of Fig. 1b with point–like quark–photon vertex. Such a treatment of low–Q^2 photon–meson form factor has been criticized in Ref. [14]. Our consideration also does not support this picture: whereas the hard photon can be treated as a standard QED point-like particle, the soft photon structure is nontrivial and is very much like the structure of other ground state mesons. This hadron-like structure of the soft photon is crucial for the description of the photon–meson transition form factors at low Q^2.

In the case when both of the photon virtualities are nonzero, our approach recovers at large Q^2 the $1/Q^2$–behavior as PQCD does, and not $1/Q_1^2 Q_2^2$ as might be expected from the naive application of the vector meson dominance. The vector meson dominance reveals itself more in the hadron-like structure of the soft photon, and not in a naive $1/Q_1^2$ behavior of the form factors.

The study of transition form factors in the region of a few GeV^2 has been considered in some papers (see Ref. [15] and references therein) as a possibility to discriminate between the pion distribution amplitudes of the Chernyak-Zhitnitsky type and the asymptotic one of Eq. (4): it was assumed that the form factor in this region can be described by the leading PQCD term which is considered within modified hard scattering picture. Our analysis shows that such an approach is not well–justified in the region of a few GeV^2: the contributiuon of the soft region is far from being small. The detailed quantitative analysises performed in Ref. [1] and in this paper as well as recent QCD sum rule results [16] show that the distribution amplitude is numerically very close to the asymptotic form of Eq. (4) and disregard the double–humped distribution amplitudes of the Chernyak–Zhitnitsky type.

V. ACKNOWLEDGEMENTS

We thank L.G.Dakhno for useful discussions and remarks. We also garateful to International Science Foundation for the financial support under the grant R10300.

APPENDIX: TRACE CALCULATIONS IN THE SOFT–HARD CONTRIBUTION

Soft–hard contribution is described by the two-loop diagram, and the dispersion relation technique prescribes that the total momenta squared of the $q\bar{q}$ pair should be taken different for each loop, and then the dispersion integration over these values should be performed. So, we must first make the Fierz rearrangements to obtain trace calculations related to different loops. Namely, we group the expressions as follows:

$$\begin{aligned} &Sp\left(i\gamma_5(m-\hat{k}_2)\gamma_\alpha(m-\hat{k}'_2)\gamma_\nu(m+\hat{k}'_1)\gamma_\alpha \right. \\ &\qquad\qquad \left. \times(m+\hat{k}''_1)\gamma_\mu(m+\hat{k}_1)\right) = Sp^{(1)}_{\mu\nu} = \\ &\sum_{i=S,V,T,A,P} C_i Sp\left((m+\hat{k}''_1)\gamma_\mu(m+\hat{k}_1)i\gamma_5(m-\hat{k}_2)O_i\right) \\ &\qquad\qquad \times Sp\left(O_i(m-\hat{k}'_2)\gamma_\nu(m+\hat{k}'_1)\right), \end{aligned} \tag{A1}$$

$$\begin{aligned} &Sp\left(i\gamma_5(m-\hat{k}_2)\gamma_\alpha(m-\hat{k}'_2)\gamma_\nu(m+\hat{k}'_1)\gamma_\mu \right. \\ &\qquad\qquad \left. \times(m+\hat{k}''_1)\gamma_\alpha(m+\hat{k}_1)\right) = Sp^{(2)}_{\mu\nu} = \\ &\sum_{i=S,V,T,A,P} C_i Sp\left((m+\hat{k}_1)i\gamma_5(m-\hat{k}_2)O_i\right) \\ &\qquad \times Sp\left(O_i(m-\hat{k}'_2)\gamma_\nu(m+\hat{k}'_1)\gamma_\mu(m+\hat{k}''_1)\right), \end{aligned} \tag{A2}$$

with $C_S = 1, C_V = -\frac{1}{2}, C_T = 0, C_A = \frac{1}{2}, C_P = -1$. Each term ($i = S,V,T,A,P$) in Eq. (A1, A2) is represented as a product of two factors, related to two different loops (see Fig. 7). In Eq. (A1) the $S-, V-, A-$terms are nonzero while in Eq. (A2) the $P-, A-$terms.

In calculations of the diagarams of Fig. 7a,b we use the following relations,

$$\begin{aligned} (\text{Fig. 7a}):\quad & P = k_1 + k_2,\ P' = k'_1 + k'_2, \\ & P'' = k''_1 + k_2,\ P'' = P + q, \end{aligned} \tag{A3}$$

$$\begin{aligned} (\text{Fig. 7b}):\quad & P = k_1 + k_2,\ P' = k'_1 + k'_2, \\ & P'' = k''_1 + k'_2,\ P' = P'' + q. \end{aligned} \tag{A4}$$

To treat the expressions

$$k_\mu,\ k'_\mu,\ k_\mu k'_\nu,\ k'_\mu k'_\nu,$$

we have to represent $k_\mu,\ k'_\mu$ in the form

$$
\begin{aligned}
k_\mu &= a_1 P_\mu + a_2 q_\mu + a_3 \delta_\mu, \\
k'_\mu &= a'_1 P_\mu + a'_2 q_\mu + a'_3 \delta_\mu, \\
& \quad q = P' - P, \\
\text{(Fig. 7a)}: & \quad \delta = P'' - P', \\
\text{(Fig. 7b)}: & \quad \delta = P'' - P, \\
a_1 &= \frac{(k\delta)}{(P\delta)} \ , \ \ a_2 = \frac{(k\delta)(Pq)}{(P\delta)Q^2} - \frac{(kq)}{Q^2}, \\
a'_1 &= \frac{(k'\delta)}{(P\delta)} \ , \ \ a'_2 = \frac{(k'\delta)(Pq)}{(P\delta)Q^2} - \frac{(k'q)}{Q^2}. \qquad \text{(A5)}
\end{aligned}
$$

After isolating invariant amplitudes for which we write down the dispersion relations over s, s' and s'', the four-vectors should be put on mass shell in the Lorentz tensor structures using the conditions $P' = P + q$ (Fig. 7a) or $P'' = P$ (Fig. 7b).

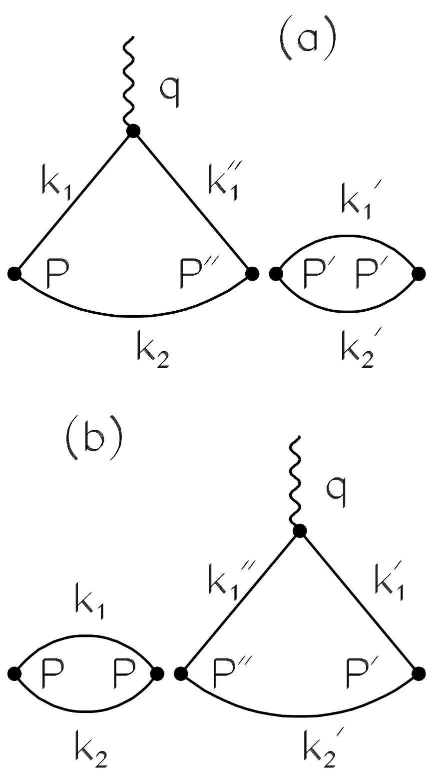


FIG. 7. Momentum notations in soft–hard terms $F^{SH(1)}$ (a) and $F^{SH(2)}$ (b).

To relate different scalar products, which appear during calculations of traces of Eq. (A1, A2), with light–cone variables we use the following four–vectors which are common for both diagrams of Fig. 7a and 7b:

$$
\begin{aligned}
k &= \left(xP_+, \frac{m^2 + \vec{k}_\perp^2}{2xP_+}, \vec{k}_\perp \right), k' = \left(x'P_+, \frac{m^2 + \vec{k}_\perp'^2}{2x'P_+}, \vec{k}'_\perp \right), \\
P' &= \left(P_+, \frac{s' + Q^2}{2P_+}, \vec{Q} \right), q = \left(0, \frac{s' - s + Q^2}{2P_+}, \vec{Q} \right), \\
P &= \left(P_+ \ , \frac{s}{2P_+}, 0 \right). \qquad \text{(A6)}
\end{aligned}
$$

and four–vectors which are different for diagrams of Fig. 7a and 7b:

$$
\begin{aligned}
\text{(7a)}: P'' &= \left(P_+, \frac{s'' + Q^2}{2P_+}, \vec{Q} \right), \delta = \left(0, \frac{s'' - s'}{2P_+}, 0 \right), \\
\text{(7b)}: P'' &= \left(P_+, \frac{s''}{2P_+}, 0 \right), \delta = \left(0, \frac{s'' - s}{2P_+}, 0 \right). \qquad \text{(A7)}
\end{aligned}
$$

With Eq. (43) (or Eq. (54)), the coefficients in Eq. (A5) are equal to:

$$
a_1 = x, \quad a'_1 = x', \quad a_2 = \frac{(\vec{k}_\perp \vec{Q})}{Q^2}, \quad a'_2 = x' + \frac{(\vec{k}'_\perp \vec{Q})}{Q^2}. \qquad \text{(A8)}
$$

Expansions of $k_\mu k'_\nu$ and $k'_\mu k'_\nu$ read as

$$
\begin{aligned}
k_\mu k'_\nu &= \sum_{A,B=P,q,\delta} a'_{AB} A_\mu B_\nu + b' g_{\mu\nu}, \\
k'_\mu k'_\nu &= \sum_{A,B=P,q,\delta} a''_{AB} A_\mu B_\nu + b'' g_{\mu\nu}, \qquad \text{(A9)}
\end{aligned}
$$

where $A_\mu = P_\mu,\ q_\mu$, or δ_μ and $B_\nu = P_\nu,\ q_\nu$, or δ_ν. For the calculation, we need coefficients b' and b'' only. For that one has to multiply both sides of Eq. (A9) by $\Pi_{\mu\nu}$, which is orthogonal to each pair $A_\mu B_\nu$, and $\Pi_{\mu\nu} g_{\mu\nu} = 1$:

$$
\begin{aligned}
\Pi_{\mu\nu} &= g_{\mu\nu} + \alpha_1 P_\mu P_\nu + \alpha_2 q_\mu q_\nu + \alpha_3 \delta_\mu \delta_\nu \\
&+ \beta_1 (P_\mu q_\nu + P_\nu q_\mu) + \beta_2 (P_\mu \delta_\nu + P_\nu \delta_\mu) \\
&+ \beta_3 (q_\mu \delta_\nu + q_\nu \delta_\mu), \qquad \text{(A10)}
\end{aligned}
$$

$$
\begin{aligned}
\alpha_1 &= \frac{(q\delta)^2 - q^2\delta^2}{D}, \quad \beta_1 = \frac{\delta^2 (Pq) - (P\delta)(q\delta)}{D}, \\
\alpha_2 &= \frac{(P\delta)^2 - P^2\delta^2}{D}, \quad \beta_2 = \frac{q^2 (P\delta) - (Pq)(q\delta)}{D}, \\
\alpha_3 &= \frac{(Pq)^2 - P^2 q^2}{D}, \quad \beta_3 = \frac{P^2 (q\delta) - (Pq)(P\delta)}{D}, \\
D &= P^2 q^2 \delta^2 + 2(Pq)(P\delta)(q\delta) \\
&\quad - P^2 (q\delta)^2 - q^2 (P\delta)^2 - \delta^2 (Pq)^2. \qquad \text{(A11)}
\end{aligned}
$$

Then,

$$
\begin{aligned}
b' &= \Pi_{\mu\nu} k_\mu k'_\nu = \frac{(\vec{k}_\perp \vec{Q})(\vec{k}'_\perp \vec{Q})}{Q^2} - (\vec{k}_\perp \vec{k}'_\perp), \\
b'' &= \Pi_{\mu\nu} k'_\mu k'_\nu = \frac{(\vec{k}'_\perp \vec{Q})^2}{Q^2} - \vec{k}_\perp'^2. \qquad \text{(A12)}
\end{aligned}
$$

Now we can write down final expressions for traces of Eqs. (A1, A2):

$$
\begin{aligned}
Sp^{(1)}_{\mu\nu} &= 4m \varepsilon_{\mu\mu\alpha\beta} q^\alpha P^\beta \big(s' + 4b'' - 12b' \\
&\quad + (s'' - s - q^2)(a'_1 - a'_2) \big), \qquad \text{(A13)}
\end{aligned}
$$

$$
\begin{aligned}
Sp^{(1)}_{\mu\nu} &= 8m \varepsilon_{\mu\mu\alpha\beta} q^\alpha P^\beta \big(s - (a'_1 - a'_2)(k''_1 P) - 2b'' \\
&\quad - a'_2 (k'_1 k'_2) - m^2 (1 + a'_2) \big), \qquad \text{(A14)}
\end{aligned}
$$

where

$$(k_1''P) = \frac{1}{2}(x's'' + (1-x')s),$$
$$(k_1'k_2') = \frac{1}{2}s' - m^2. \qquad \text{(A15)}$$

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TABLE I. Step–function approximation for soft wave functions ($\vec{k}^2$ is given for the middle point of each step of Fig. 3). All quantities are in GeV.

$\vec{k}^2$	$\psi_{n\bar{n}}$	$\psi_{s\bar{s}}$	$\psi_{\gamma\to n\bar{n}}$	$\psi_{\gamma\to s\bar{s}}$
0.033	87.737	42.316	14.488	7.949
0.096	48.591	27.154	7.700	4.864
0.159	28.170	17.194	5.006	3.445
0.221	15.385	9.972	2.881	2.102
0.284	9.005	6.096	2.438	1.855
0.346	4.917	3.440	1.864	1.465
0.440	2.216	1.609	1.369	1.116
0.565	1.610	1.210	0.941	0.794
0.690	1.480	1.140	0.608	0.526
0.815	1.592	1.249	0.575	0.506
0.940	1.905	1.517	0.602	0.538
1.065	1.974	1.589	0.548	0.495
1.190	2.227	1.810	0.560	0.510
1.315	2.477	2.029	0.531	0.488
1.440	2.307	1.902	0.516	0.477
1.565	2.489	2.063	0.479	0.445
1.690	2.029	1.690	0.473	0.442
1.815	1.687	1.412	0.444	0.417
1.940	1.525	1.281	0.446	0.420
2.065	1.163	0.981	0.453	0.428